Chapter 3

Section 3.1 - Derivative of a Function

From section 2.4, recall that

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

When this limit exists, it is called the

DERIVATIVE of f at a

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Derivative is

Slope of the curve
Slope of the tangent
Instaneous Rate of Change (ROC)
at a point

$f'(x) = y'$

$\frac{dy}{dx}$ → **The derivative of y with respect to x**

$\frac{d}{dx} f(x)$

Notation

There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are these:

y'	"y prime"	Nice and brief, but does not name the independent variable.
$\frac{dy}{dx}$	"*dy dx*" or "the derivative of *y* with respect to *x*"	Names both variables and uses *d* for derivative.
$\frac{df}{dx}$	"*df dx*" or "the derivative of *f* with respect to *x*"	Emphasizes the function's name.
$\frac{d}{dx}f(x)$	"*d dx* of *f* at *x*" or "the derivative of *f* at *x*"	Emphasizes the idea that differentiation is an operation performed on *f*.

Example

Differentiate (find the derivative of)

$f(x) = x^3$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x+h)(x+h) - x^3}{h}$$

$$= \lim_{h \to 0} \frac{(x^2+2xh+h^2)(x+h) - x^3}{h}$$

Find the derivative of $f(x) = \frac{1}{x}$

$\frac{-h}{h}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x-x-h}{x(x+h)}}{\frac{h}{1}}$$

$$\lim_{h \to 0} \frac{1(\cancel{x-x}-h)}{xh(x+h)} \quad \frac{-\cancel{h}}{x\cancel{h}(x+h)}$$

$$f'(2) = \frac{-1}{2^2} = \frac{-1}{4}$$

Alternate Definition (Makes your life easier)

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example

Differentiate f(x) = $\sqrt{x}$ using the alternate definition

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sqrt{x} - \sqrt{a}}{x - a} \cdot \frac{\sqrt{x} + \sqrt{a}}{\sqrt{x} + \sqrt{a}}$$

$$= \lim_{x \to a} \frac{x - a}{(x-a)(\sqrt{x} + \sqrt{a})} = \lim_{x \to a} \frac{1}{\sqrt{x} + \sqrt{a}} = \frac{1}{\sqrt{a} + \sqrt{a}} = \frac{1}{2\sqrt{a}}$$

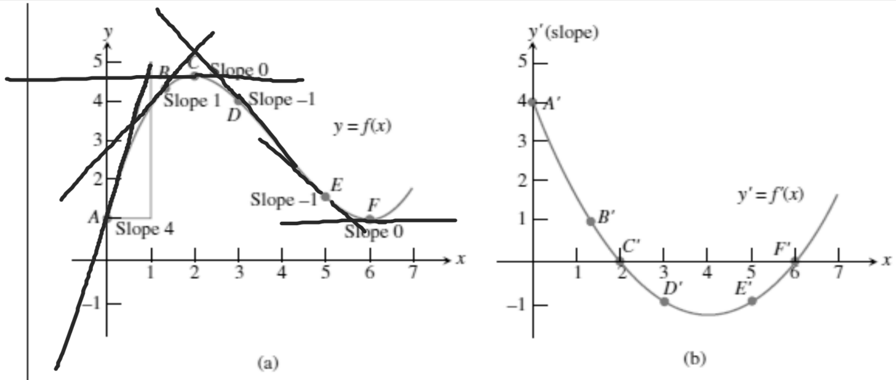

Figure 3.3 By plotting the slopes at points on the graph of $y = f(x)$, we obtain a graph of $y' = f'(x)$. The slope at point A of the graph of f in part (a) is the y-coordinate of point A' on the graph of f' in part (b), and so on. (Example 3)

One-Sided Derivatives

A function y = f(x) is differentiable on a closed interval [a,b] if it has a derivative at every interior point of the interval, and if the limits

$$\lim_{h\to 0^+} \frac{f(a+h) - f(a)}{h} \quad \text{[the right hand derivative at a]}$$

$$\lim_{h\to 0^-} \frac{f(b+h) - f(b)}{h} \quad \text{[the left hand derivative at b]}$$

exist at the endpoints

Example

Show that the following function has left-hand and right-hand derivatives at x = 0, but no derivative there

$$y = \begin{cases} x^2 & x \leq 0 \\ 2x & x > 0 \end{cases}$$

$2x$

$$\lim_{h\to 0^+} \frac{f(0+h) - f(0)}{h} = \lim_{h\to 0^+} \frac{2h - 0}{h} = 2$$

x^2

$$\lim_{h\to 0^-} \frac{h^2 - 0}{h} = \lim_{h\to 0} h = 0$$

Page 105, numbers 1 - 12 all,17,18,19